On the Calculation of Boundary Layers along Rotating Cylinders

J. S. Tennant*
Florida Atlantic University, Boca Raton, Fla.

W. S. Johnson†
University of Tennessee, Knoxville, Tenn.

and

D.D. Keaton‡
General Dynamics Corporation, Fort Worth, Tex.

Nomenclature

P = static pressure
r = cylinder radius
x = boundary-layer coordinate measured from stagnation point
y = boundary-layer coordinate normal to surface
U = local x-direction velocity in the boundary layer
U_e = local velocity at the edge of the boundary layer
U_∞ = freestream velocity
V = local y-direction velocity in the boundary layer
β = ratio of cylinder surface speed to freestream velocity
δ = local boundary-layer thickness
η = similarity parameter $(a/\nu)^{1/2}y$
θ = angle measured from stagnation point
ρ = fluid density
ν = fluid kinematic viscosity

Received Aug. 12, 1976.
Index category: Boundary-Layers and Convective Heat Transfer-Laminar.

*Associate Professor of Ocean Engineering.
†Associate Professor of Mechanical and Aerospace Engineering.
‡Research Engineer.

Introduction

THE calculation of a boundary layer on a rotating cylinder in crossflow by finite difference techniques requires the specification of an initial velocity profile near the stagnation point and a moving surface boundary condition along with the streamwise pressure distribution. General solutions for this problem are quite complex, however, due to the unsymmetrical wake region which produces a strong coupling between the boundary layer and potential flowfield.

The objective of this Note is to present the successful application of a well-known finite difference boundary-layer procedure to the rotating cylinder problem incorporating an exact calculation of the initial profiles based on stagnation flow onto a moving wall. The comparison is only made for the sied of the cylinders moving streamwise.

Initial Profile

Consider stagnation flow onto a moving surface as shown in Fig. 1. The following analysis is a modification of the fixed wall stagnation flow analysis due to Himenez and reported in Schlicting.[1]

The x-direction momentum equation

$$U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}\right)$$

and the 2-dimensional continuity equation

$$(\partial U/\partial x) + (\partial V/\partial y) = 0$$

may be reduced to an ordinary differential equation by a similarity transformation first introduced by Glauert[2] for the solution of the boundary layer in the stagnation region of a cylinder oscillating about its axis.

The similarity transformation is

$$U = xf'(y) + g(y)$$

and

$$V = -f(y)$$

which assumes that the V component of velocity is unaffected by wall motion and that the effect on the U component is independent of x. These variables satisfy continuity identically. Substitution of the similarity variables into the momentum equation and expressing the pressure gradient as

$$(dP/dx) = -\rho a^2 x$$

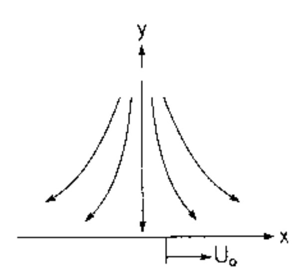

Fig. 1 Stagnation flow onto a moving wall.

from a potential flow analysis where a is a constant yields

$$-x[f'^2 - ff'' - a^2 - \nu f'''] = gf' - fg' - \nu g'' \qquad (1)$$

Noting that the right-hand side of Eq. (1) is independent of x, the term in brackets must equal zero. Thus

$$f'^2 - ff'' - a^2 - \nu f''' = 0 \qquad (2)$$

Differentiating, gives

$$f'f'' - ff''' - \nu f'''' = 0$$

Comparison with the right-hand side of Eq. (1) indicates that

$$g = C_l f''$$

where C_l is an undetermined constant. A further transformation to simplify Eq. (2) is

$$f(y) = (a\nu)^{1/2}\phi(\eta)$$

where $\eta = (a/\nu)^{1/2} y$ results in the equation

$$\phi''' + \phi\phi'' - \phi'^2 + 1 = 0$$

which is the same equation solved for the fixed wall case and the solution is reported in Schlicting and elsewhere. The velocity components become

$$U = xa\phi'(\eta) + C_l a (a/\nu)^{1/2}\phi''(\eta)$$

and

$$V = -(a/\nu)^{1/2}\phi(\eta)$$

The constants a and C_l may be found by matching this solution to the potential flow solution along with the application of appropriate wall boundary conditions to obtain

$$\frac{U}{U_\infty} = \frac{2x}{r}\phi'(\eta) + \frac{\beta}{1.2326}\phi''(\eta)$$

and

$$\frac{V}{U_\infty} = -\left(\frac{2}{U_\infty r\nu}\right)^{1/2}\phi(\eta)$$

These equations may be used to calculate the initial conditions at any location near the stagnation point.

Calculations and Comparison with Experiment

The difficulty in obtaining measured velocity profiles near a rotating cylinder is borne out by the scarcity of data available in the literature. Brady and Ludwig[3] did, however, obtain boundary-layer velocity profiles on a 4 in. rotating cylinder while studying fundamental characteristics of stall. Their configuration consisted of a rotating cylinder with shrouds as shown in Fig. 2. A measured pressure distribution along the cylinder surface was obtained with a rotating static port in the cylinder surface that was connected to a transducer. This pressure distribution is given in Fig. 3.

With the starting profile as previously described and the measured pressure distribution from Fig. 3, a modified version of the Cebeci-Smith[4] finite difference boundary-layer calculation technique was applied. This technique has been previously modified by Tennant[5] for a moving wall boundary condition in laminar or turbulent flow, the latter requiring an additional modification to the eddy viscosity. However, the boundary layer in this problem was entirely laminar as determined by the Reynolds Number based on momentum thickness, which remained below 360 and this is an accepted criteria for transition on a fixed wall. For the moving wall case, Swanson[6] reported that transition to turbulent boundary layer is retarded relative to the fixed wall case on the side of the cylinder moving streamwise.

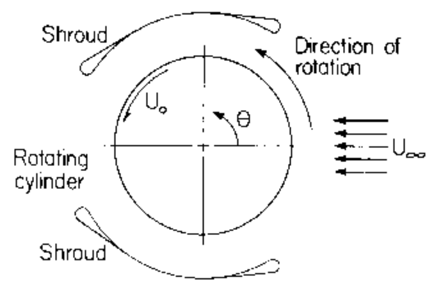

Fig. 2 Test configuration – REF. 3

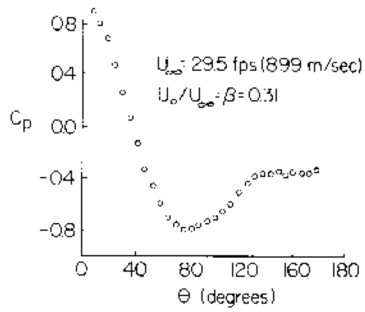

Fig. 3 Surface pressure distribution REF. 3

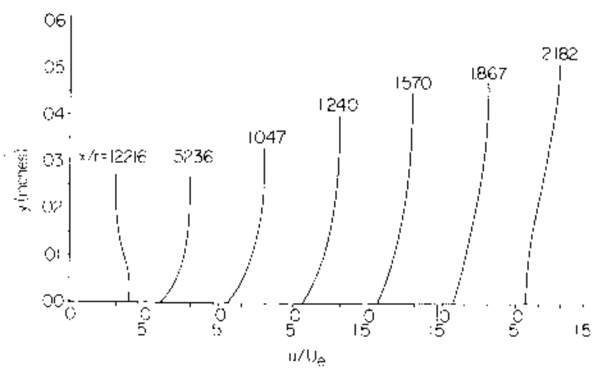

Fig. 4 Velocity profile development.

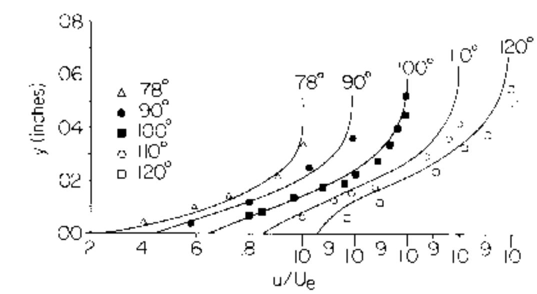

Fig. 5 Comparison of predicted and measured velocity profiles.

Figure 4 presents the development of the velocity profile from the leading edge of the cylinder to the separation point as predicted by the analysis for $\beta = 0.30$. The initial profile is taken from the leading edge similarity solution obtained at $x/r = 0.12216$ ($\theta - 7°$). Figure 5 shows a comparison of predicted and measured profiles at $\theta = 78°$, 90°, 100°, 110°, and 120° for a freestream velocity of 9.0 m/sec and a cylinder surface speed of 3.0 m/sec. The agreement is seen to be excellent at all locations except 120° where proximity to the separation point produces the traditional difficulty in solving the boundary-layer equations in this region.

Conclusions

Based on this study it may be concluded that the Glauert transformation results in a similarity solution which produces a satisfactory starting velocity profile for the numerical calculation procedure. The calculation procedure based on a modification of the Cebeci-Smith method accurately predicts the boundary-layer development on a rotating cylinder for the case of a relatively low wall velocity on the streamwise moving side. No data is available to determine the accuracy of this procedure at high wall velocities.

Acknowledgment

This work was sponsored by the U.S. Navy Ship Research and Development Center, Contract No. N00014-71-A-0121-006.

References

[1]Schlicting, H., *Boundary Layer Theory*, McGraw Hill, 1968.

[2]Glauert, M.B., "The Laminar Boundary Layer on Oscillating Plates and Cylinders," *Journal of Fluid Mechanics*, Vol. 1, 1956, p. 97.

[3]Brady, W. G. and Ludwig, G. R., "Basic Studies of Rotating Stall and Investigation of Flow-Instability Sensing Devices-Part 1," Rept. APAPL-7R-65-115, Part 1, Air Force Aero-Propulsion Lab, Wright-Patterson AFB, Oct. 1965.

[4]Cebeci, T. and Smith, A.M.O., "A Finite Difference Solution of the Incompressible Turbulent Boundary Layer Equations by an Eddy Viscosity Concept," Rept. DAC-67130, McDonnel Aircraft Co., Long Beach Calif., Oct. 1968.

[5]Tennant, J. S. and Yang, T., "Turbulent Boundary Layer Flows from Stationary to Moving Surfaces," *AIAA Journal*, Vol. 11, Aug. 1973, pp. 1156-1161.

[6]Swanson, W.M., "The Magnus Effect: A Summary of Investigations to Date," *ASME Journal of Basic Engineering*, Vol. 83, Sept. 1961, pp. 461-470.